INTERNAL LETTER

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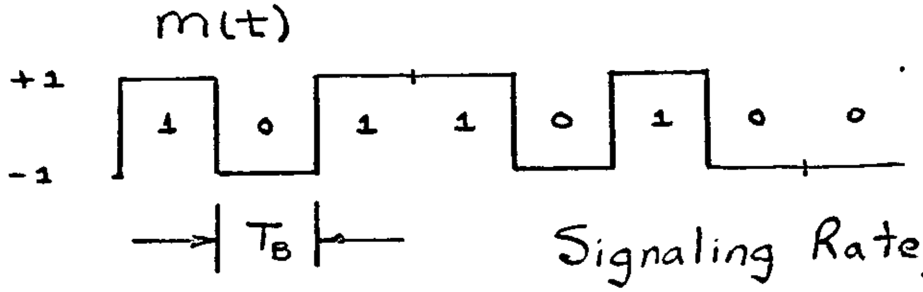
Subject: Digital Modulation Techniques — A Tutorial Presentation

This report is a brief presentation of the basic characteristics of the most commonly used digital modulation techniques. Emphasis will be given to Coherent Frequency Shift Keying (CFSK) because it embodies the features of the others and is a widely used practical technique. Binary signaling can be accomplished by modulating the amplitude, frequency, or phase of an RF carrier. In some cases, such a CFSK, control over all three is needed.

To begin, we will define the baseband binary modulating signal as having an amplitude of either +1 or -1. The amplitude may change only at time intervals corresponding to T_B, the length of a "chip" (or the time between possible amplitude transitions). Figure 1 shows a short modulating signal for the binary sequence 10110100. The modulating signal must hold a given state (±1) for at least T_B seconds or multiples thereof. The signaling rate is defined as $f_B = 1/T_B$ binary-digits-per-second (or BPS).



Signaling Rate, $f_B = \frac{1}{T_B}$ BPS

Figure 1. Binary Modulating Signal.

The RF carrier to be modulated is represented by the equation;

$$\text{RF Carrier} = A \sin [2\pi f_o t + \theta]$$

where A is the amplitude, f_o is the frequency and θ is the reference phase. With these definitions we can now define the modulation techniques.

On-Off keying or amplitude-shift keying (ASK) is characterized by the equation:

$$\text{Signal} = \frac{1}{2}[1 + m(t)] A \sin [2\pi f_o t + \theta]$$

A typical waveform for ASK is shown in Figure 2(d).

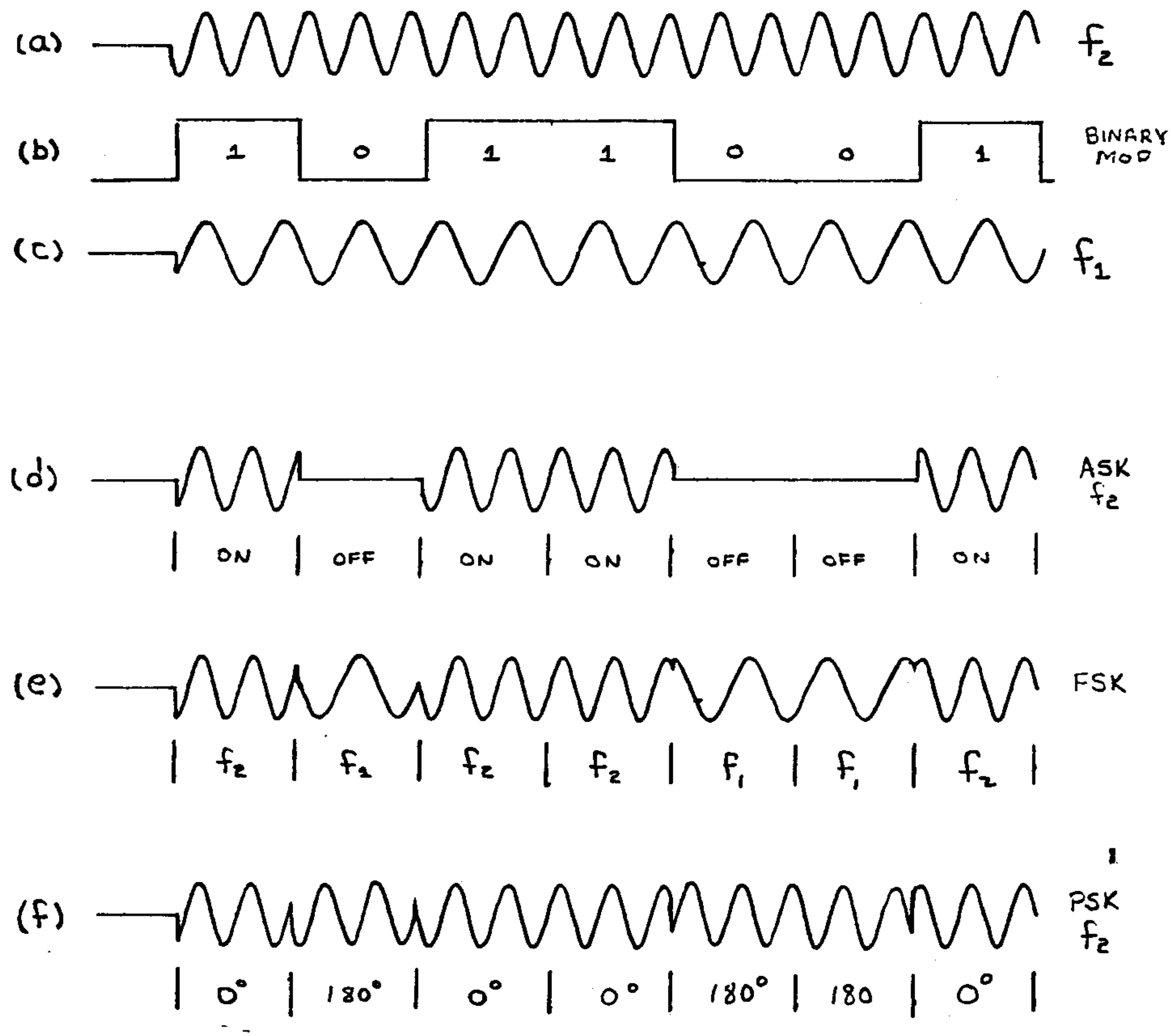


Figure 2. Carrier Waveforms for Simple ASK, FSK, and PSK Modulation.

Frequency-shift keying (FSK) is characterized by the equation:

$$\text{Signal} = A \sin\left[2\pi(f_o + m(t) f_d)t + \Theta\right]$$

Only two frequencies are transmitted, $(f_o + f_d)$ when $m(t) = +1$ and $(f_o - f_d)$ when $m(t) = -1$. An example of FSK is shown in Figure 2(e) where $f_1 = (f_o - f_d)$ and $f_2 = (f_o + f_d)$.

Phase-shift keying (PSK) is characterized by the equation:

$$\text{Signal} = A \sin\left[2\pi f_o t + \tfrac{1}{2}(1 + m(t))\pi + \Theta\right]$$

The carrier phase is switched between 0 and π at the rate determined by $m(t)$. An example of PSK is shown in Figure 2(f).

These simple modulation schemes do not take into account some important practical aspects of modulator design. These are:

1) The relationship of the carrier center frequency, f_o, to the signaling rate, $1/T_B$.

2) The relationship of the carrier phase to the bit transitions.

3) The phase coherence of the carrier from bit to bit.

These factors affect the spectrum of the transmitted signal, the design of high-power transmitter stages, and the design of frequency or phase detectors.

The first restriction we may wish to impose is to require that the bit transitions occur synchronously with either the peaks or the zero crossings of the two waveforms. Such a system for zero crossings is shown in Figure 3.

To meet such a requirement it is necessary that a bit interval, T_B, be a multiple of one cycle of f_1 and one-and-a-half cycles of f_2. This may be stated as the following requirements on frequencies and phases:

1. $f_1 = \dfrac{n}{T_B}$ $\qquad \theta_1 = +\frac{\pi}{2}, -\frac{\pi}{2}$ (Peaks)
$\qquad \theta_1 = 0, \pi$ (Zeros)

2. $f_2 = \dfrac{n+1}{T_B}$ $\qquad \theta_2 = +\frac{\pi}{2}, -\pi/2$ (Peaks)
$\qquad \theta_2 = 0, \pi$ (Zeros)

These relationships produce the nice "textbook" waveforms shown in the numerous references on digital modulation techniques. One might ask if there are

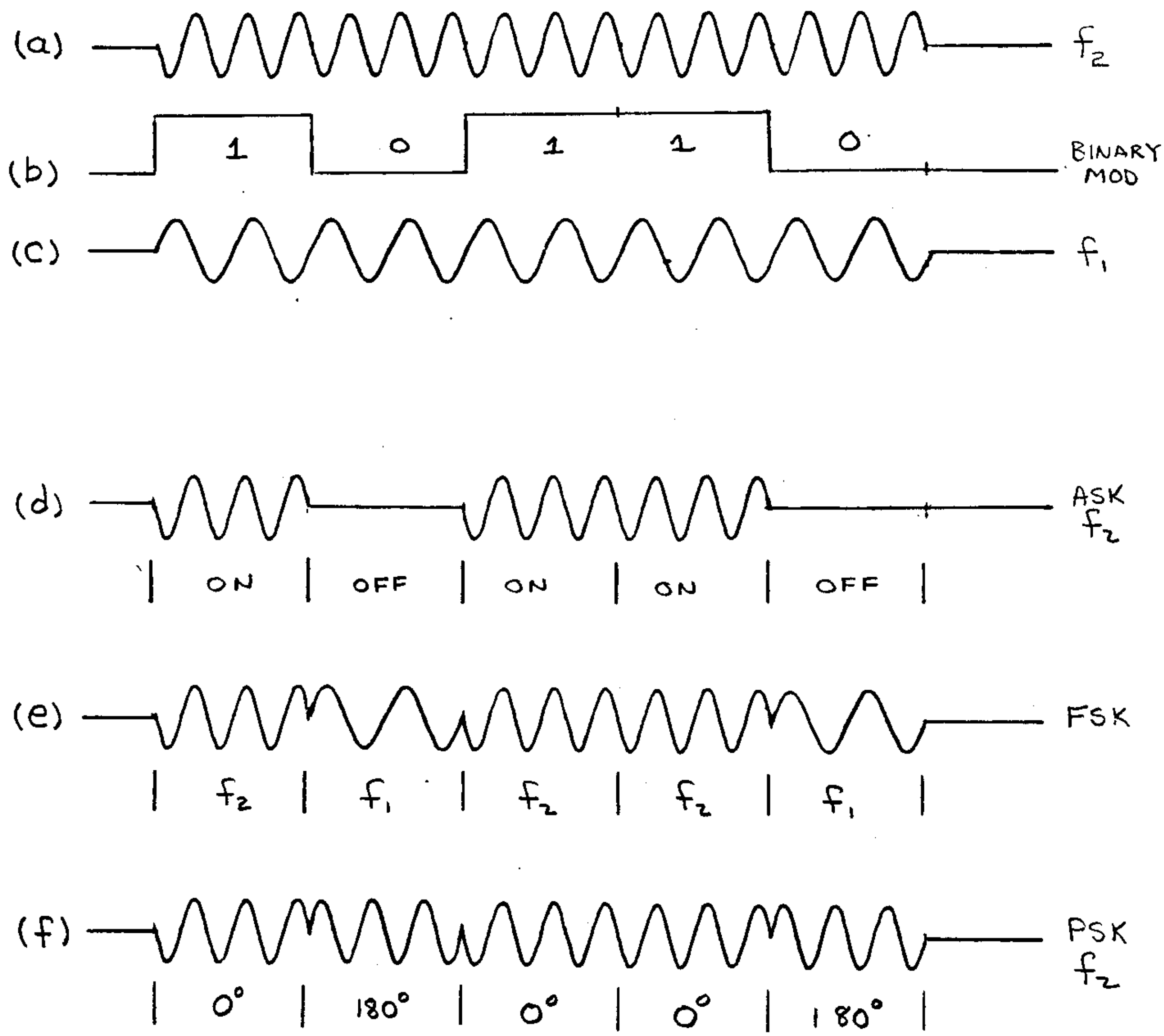
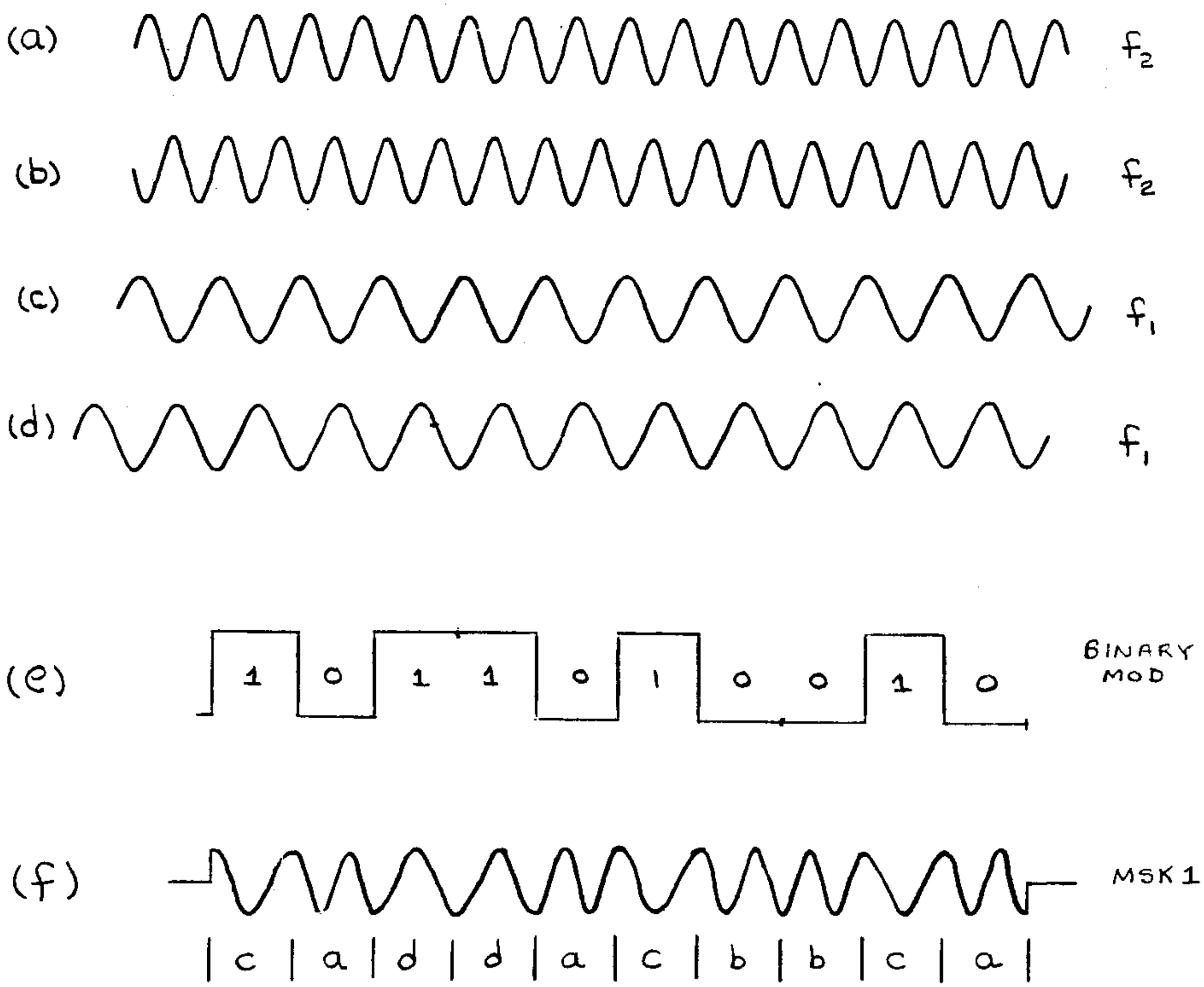


Figure 3. Carrier Waveforms for ASK, FSK, and PSK Modulation where Bit Transitions occur at the Zero Crossings of f_1 and f_2 ($n=2$).

any advantages to maintaining these restrictions and the answer is, some, but not a whole lot. Restricting the modulation change to occuring only when the carrier is peak or zero will slightly reduce the high order harmonics. This scheme does keep the phase of each frequency component coherent from interval to interval. These types of systems can be tracked with a simple phase-locked loop and do not require squaring loops or Costas loops. This is not true for the next cases we will consider.

We will restrict the rest of this discussion to a special class of FSK commonly called Minimum-Shift Keying (MSK). Implementation of MSK places unique restrictions on carrier frequency and phase but the payoff is better system performance such as reduced sidebands and the elimination of huge current transients at the bit transitions in large transmitters. MSK has also been called Compatible-Shift Keying (CSK), staggered quadraphase PSK (SQPSK), and orthogonal PSK.

Figures 4 and 5 show the waveforms associated with MSK. For reference, these will be called MSK1 and MSK2. The first thing obvious about MSK is that four different waveforms are required to generate it. The difference in the two types of MSK is that the bit transitions occur at the peak of the sinewave for MSK1 and at the zero-crossing for MSK2. The equations



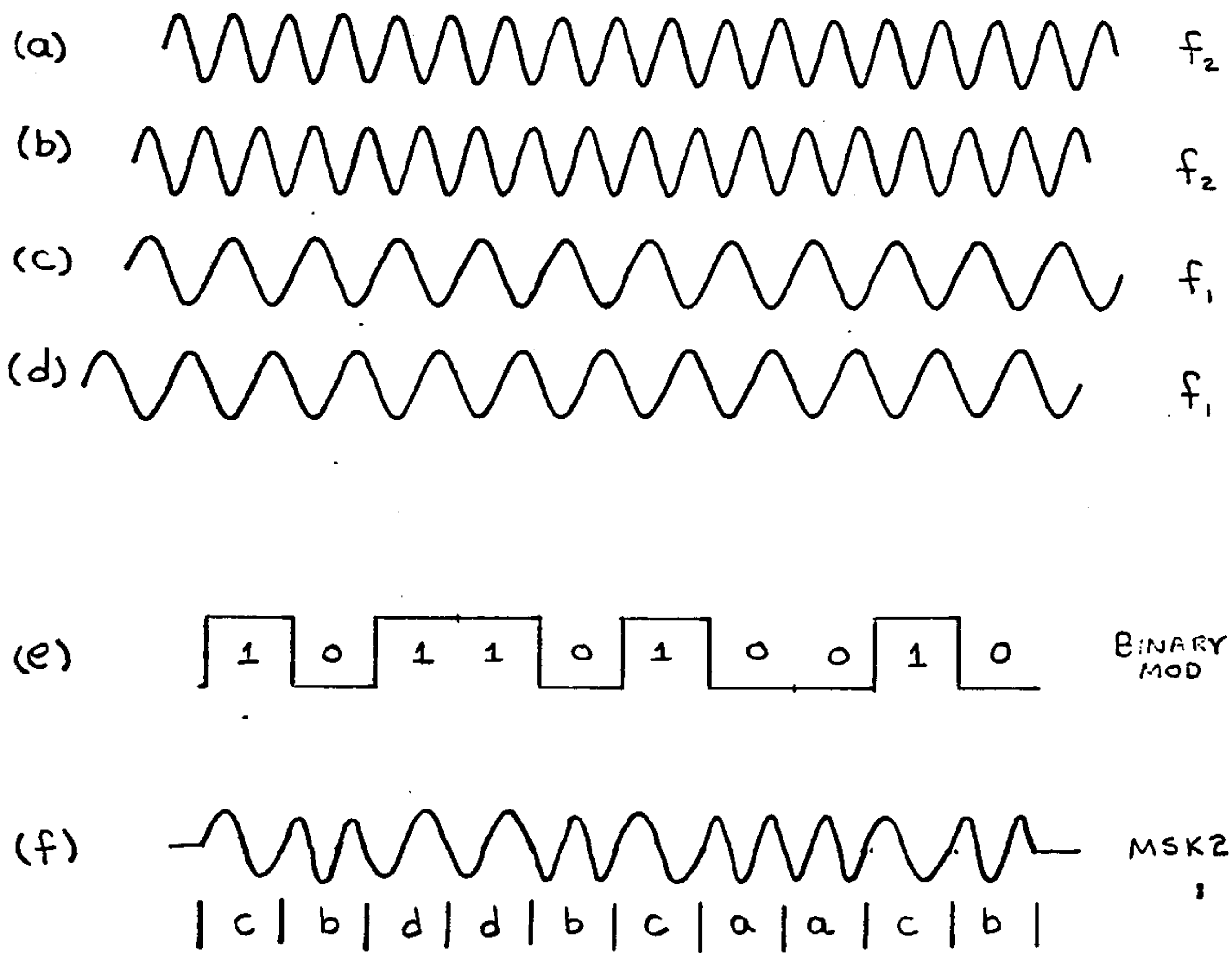
a) $A \sin [2\pi f_2 t - \frac{\pi}{2}]$

b) $A \sin [2\pi f_2 t + \frac{\pi}{2}]$

c) $A \sin [2\pi f_1 t + \frac{\pi}{2}]$

d) $A \sin [2\pi f_1 t - \frac{\pi}{2}]$

Figure 4. MSK1 Modulation Waveforms. CFSK where Bit Transitions Always Occur at Sinewave Peaks (Zero Slope).



a) $A \sin[2\pi f_2 t]$ c) $A \sin[2\pi f_1 t]$

b) $A \sin[2\pi f_2 t + \pi]$ d) $A \sin[2\pi f_1 t + \pi]$

Figure 5. MSK2 Modulation Waveforms. CFSK where Bit Transitions Always Occur at Sinewave Zero-Crossings (Zero Amplitude).

for the available modulator waveforms are given in the figures. Carrier phase is determined relative to a bit transition. Figure 6 shows the four waveforms and their relative phases over one bit interval for MSK1 and MSK2. We will now look at the characteristics of MSK in detail.

The carrier center frequency and upper and lower frequencies are related to the binary signaling rate as follows;

$$f_0 = \frac{n+1/2}{2} f_B = \frac{n+\frac{1}{2}}{2T_B} \quad \text{Hz}$$

$$f_2 = \frac{n+1}{2} f_B = \frac{n+1}{2T_B} \quad \text{Hz}$$

$$f_1 = \frac{n}{2} f_B = \frac{n}{2T_B} \quad \text{Hz}$$

where n is usually a large integer. The binary signaling rate is;

$$f_B = 2(f_2 - f_1) = \frac{1}{T_B} \quad \text{BPS}$$

The frequency separation of f_1 and f_2 is always one-half the bit rate.

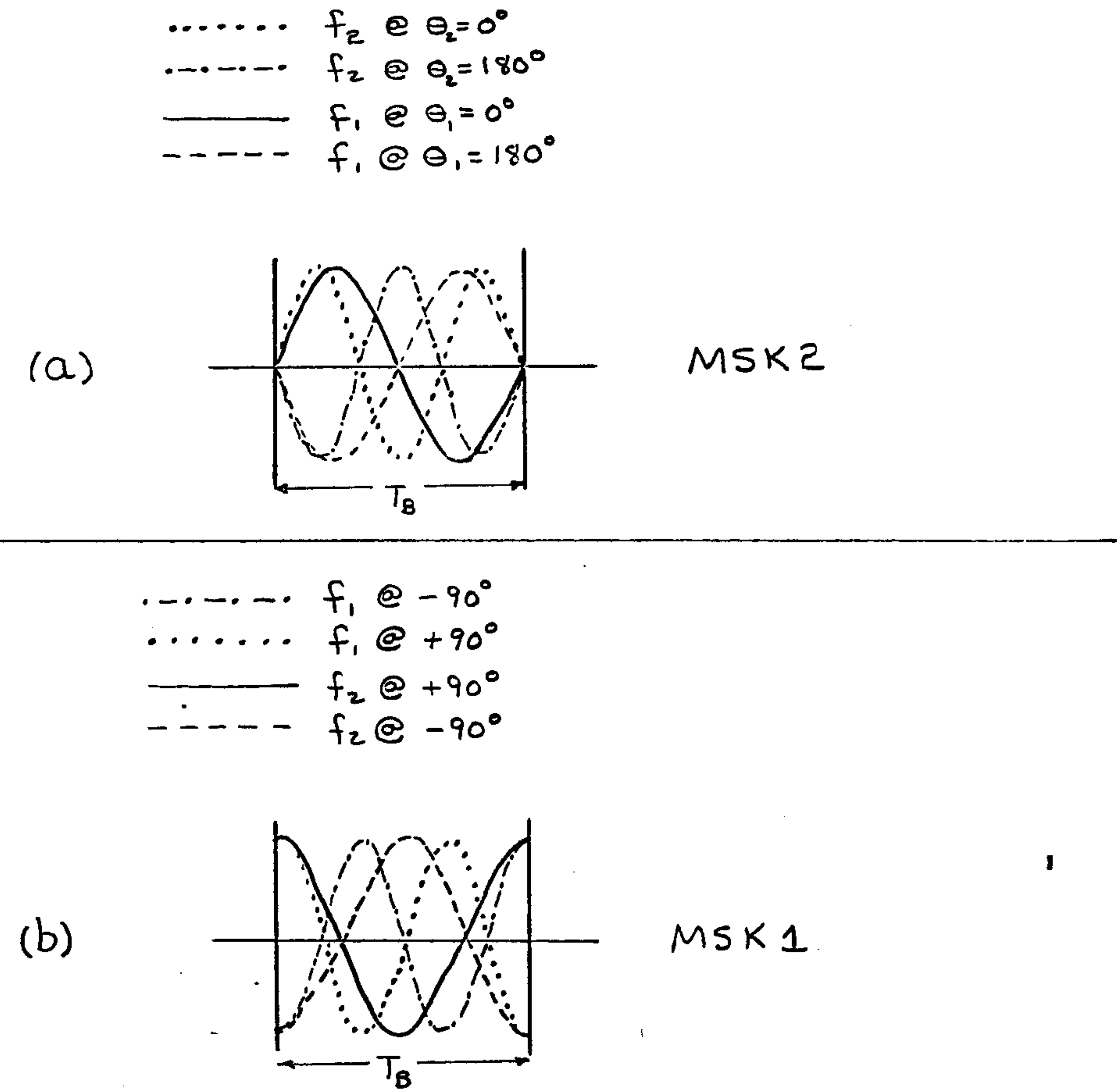


Figure 6. Available Modulator Waveforms for both types of MSK over one Bit Interval.

Figure 7 shows the frequency relationships for MSK modulation at a rate of 100 BPS. We see that the center frequency, f_0, must be multiples of 50 Hz plus an additional 25 Hz.

Another important characteristic of MSK is the lack of phase continuity that results from having to select the inphase or out-of-phase component of the carrier at any particular bit transition. This can be seen by a careful examination of Figure 5(f). If we plot only the signal segments corresponding to f_1 we obtain Figure 8. Notice the phase reversal in bit intervals 3 and 4. Actually, the sequence of ones and zeros is random so the waveform of either f_1 or f_2 looks like biphase modulation or Binary PSK (BPSK) with "gaps". We can reach the following conclusion about MSK:

With MSK, there is no carrier component at f_1, f_2, or f_0. Consequently, carrier reconstruction must be accomplished by squaring loops or Costas demodulators.

An interesting property of MSK is the path that the phase takes to achieve minimum discontinuity. Figure 9 is a copy of the resulting phase "trellis" out of Spilker (1977, P321).

$f_B = 100$ BPS $\quad T_B = 0.01$ Seconds

$(f_2 - f_1) = 50$ Hz $\quad f_0 = (50n + 25)$ Hz

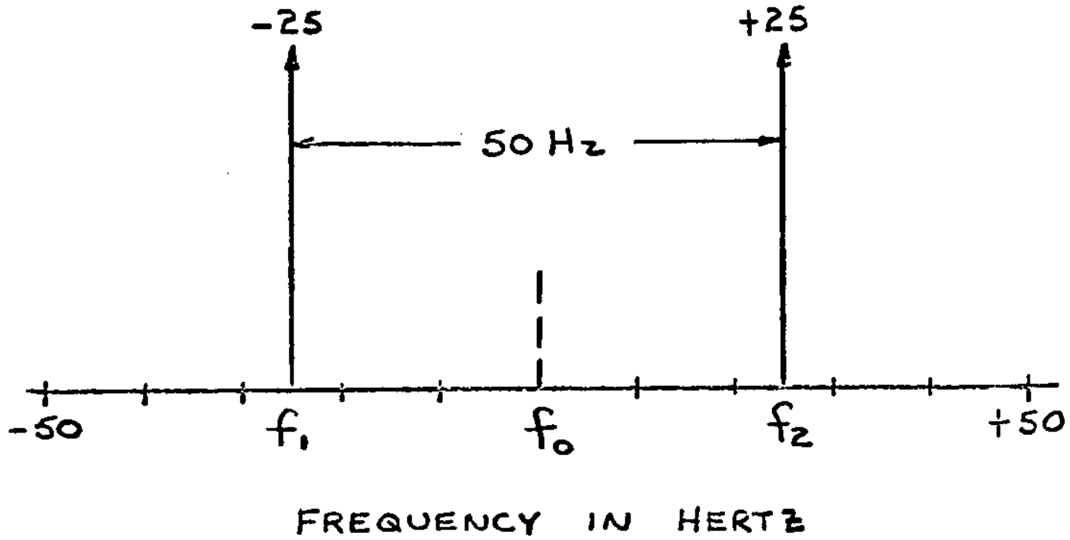


Figure 7. Frequency Relationships for MSK Modulation at a 100 BPS Rate.

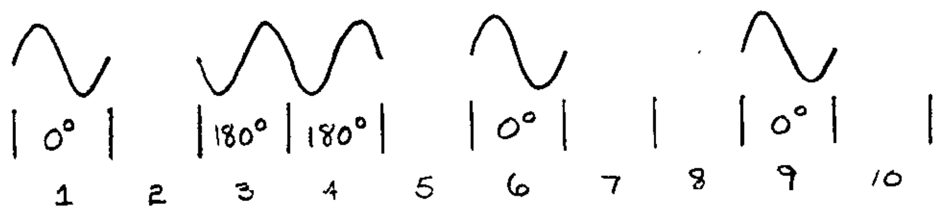


Figure 8. Carrier Segments of the f_1 Component of the Waveform of Figure 5(f) showing the Random Phase Characteristic Similiar to BPSK.

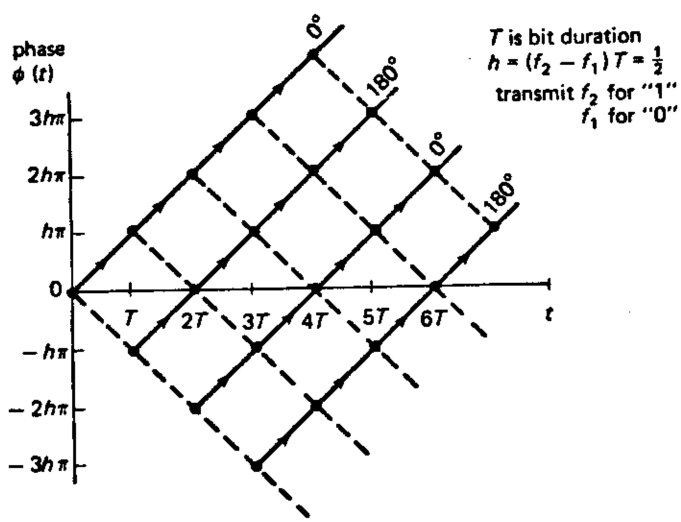


Fig. 11-22 Phase modulation for MSK; solid lines correspond to a binary "1" and the dashed lines for a binary "0".

Figure 9. Phase Trellis for MSK Modulation. (Spilker, 1977, P321)

The modulators and demodulators required to implement MSK in a system are fairly complicated when compared to simple PM or FM systems. Not only are several stages required, but various synchronous signals with known phase relationships must be generated. Figure 10 shows one implementation of an MSK modulator. Other configurations are also possible such as a SQPSK modulator. Mathematically they must all produce the same output but the order in which the processing is done may vary. A brief description of the modulator follows:

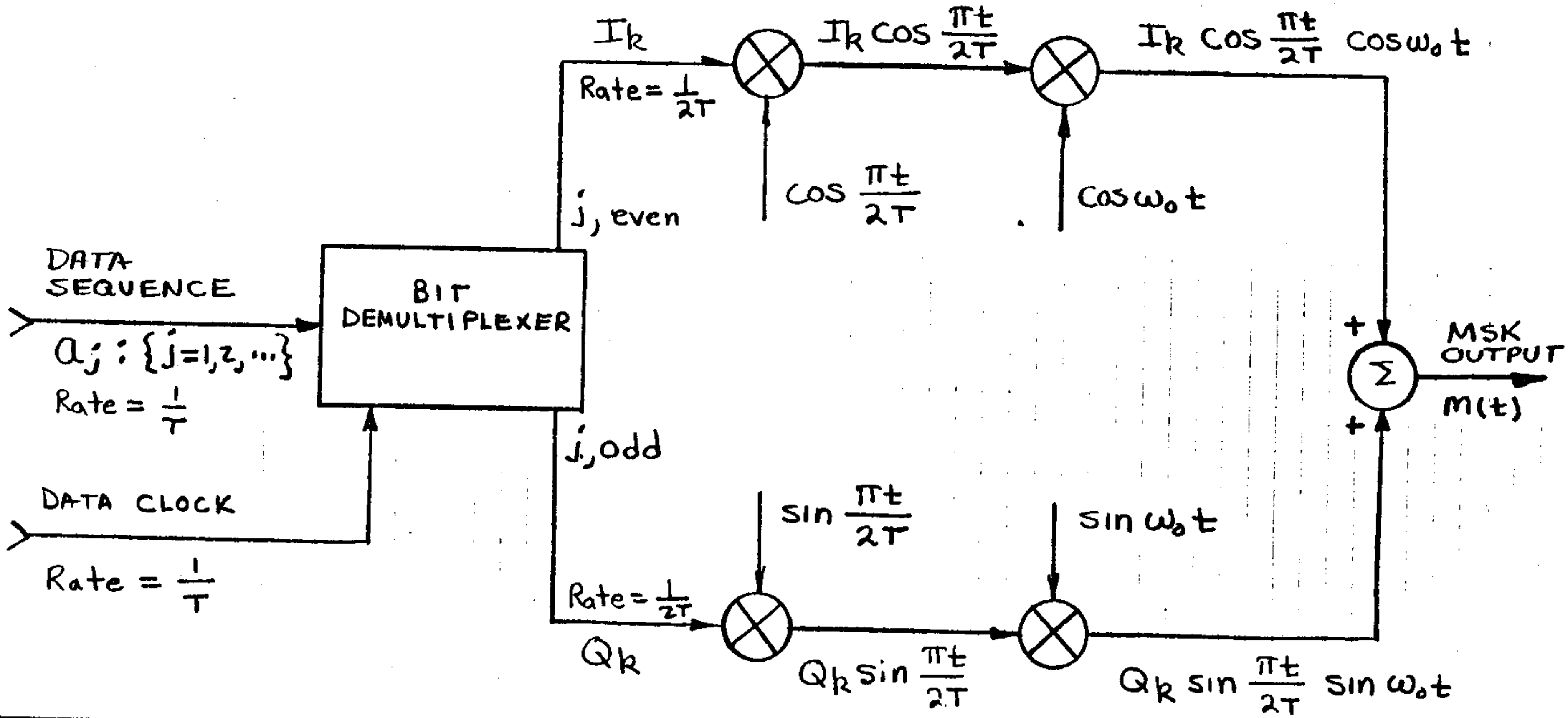
1) The data sequence consisting of a string of binary digits is input to a bit demultiplexer in groups called words. Word synchronization is accomplished by sending synchronization preambles.

2) The bit demultiplexer alternates the bits between the I and Q channels, in a sample-and-hold fashion. For example, all the bits in even numbered time slots are sent to the I-channel and the odds are sent to the Q-channel

3) The bit streams coming out of the I and Q ports of the demultiplexer have a rate of one-half of the input and twice the length.

Figure 10.

MSK MODULATOR



$I_k = a_j \quad k=j\ ,\ k \text{ even}$
$\qquad\quad k=j-1\ ,\ k \text{ odd}$

$Q_k = a_j \quad k=j\ ,\ k \text{ odd}$
$\qquad\quad k=j-1,\ k \text{ even}$

$\left.\begin{array}{l} I_1 = a_0 \\ U_0 = 1 \end{array}\right\}$ Initial Conditions

$$m(t) = I_k \cos \frac{\pi t}{2T} \cos \omega_o t + Q_k \sin \frac{\pi t}{2T} \sin \omega_o t$$
$$= I_k \cos \left[\omega_o t + (I_k \oplus Q_k) \frac{\pi t}{2T}\right]$$

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4) The bit streams are sin or cosine weighted and multiplied by inphase or quadrature components of the carrier and then summed to produce the MSK output.

All of the equations describing the modulator are shown in the figure. They will not be described in detail in this report.

The detector for MSK receivers is shown in Figure 11 from Spilker (1977, P322). This circuit performs the following functions:

1) Carrier recovery of the BPSK Carriers at f_1 and f_2. A squaring-loop implementation is used for each.

2) Bit synchronization using integrate-and-dump circuits as matched filters.

3) Data demodulation using matched filters and bit-synchronized sampling.

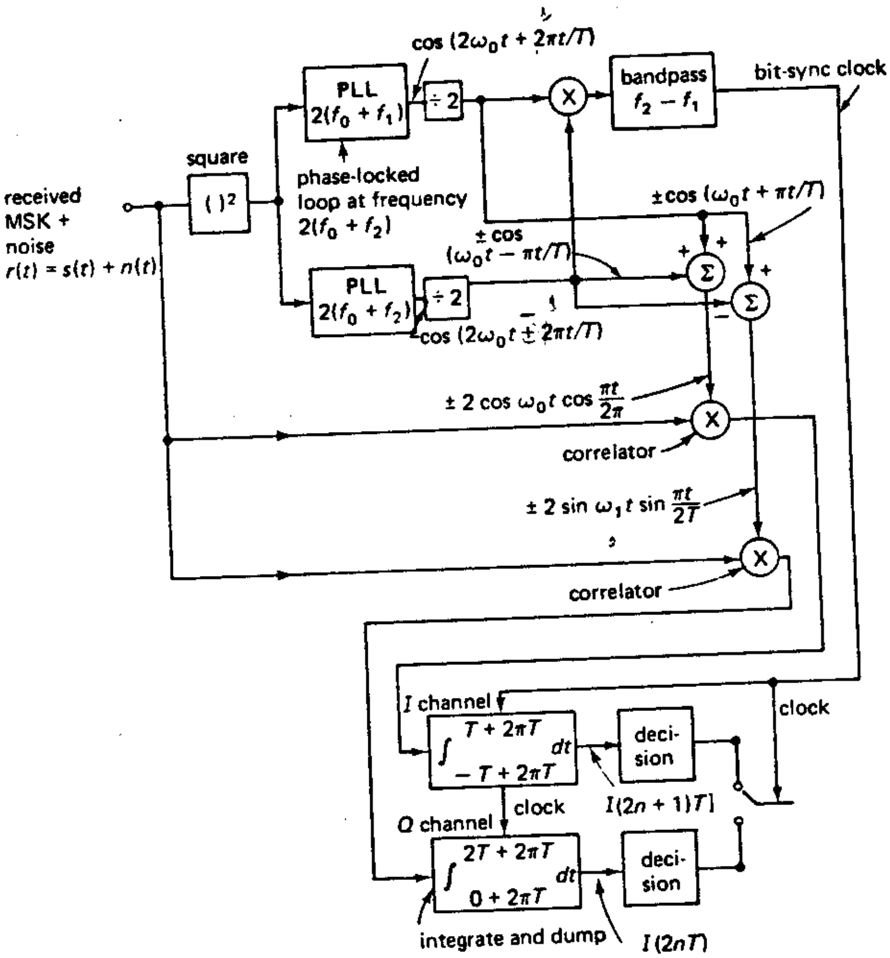

Fig. 11-23 Block diagram of an MSK receiver showing phase-locked recovery of the two BPSK carriers, clock recovery, and bit decisions made by integrating over two consecutive bits using alternate integrate-and-dump circuits. The I and Q channels can be as shown or reversed, depending on the sign ambiguity in the carrier recovery. The frequency difference $f_1 - f_2 = \frac{1}{2}T$ for $h = \frac{1}{2}$.

Figure 11. MSK Demodulator. (Spilker, 1977, P322)

The last thing to be considered is the frequency spectrums associated with some of the forms of digital modulation. They all have the characteristic $[\sin x / x]^2$ form because that is the spectral shape of the autocorrelation function of the baseband binary modulation. Figure 12 from Spilker (1977, P299) shows the spectrum for BPSK. It is the Fourier transform of a single trapezoidal pulse. Figure 13 from Mathwich et. al. (1979) shows the spectral shape of MSK both unfiltered and filtered. Figure 14 shows the integrated power spectra for BPSK, QPSK, and MSK. Note that the sidebands of MSK drop off rapidly beyond a bandwidth corresponding to the signaling rate. This result occurs because of the smooth phase changes at bit transitions.

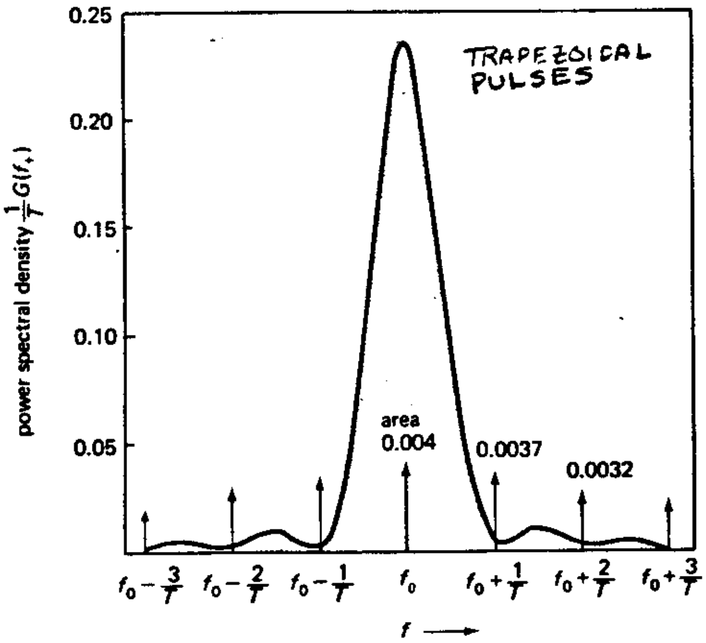

FIG 12. Power spectral densities for BPSK with trapezoidal pulses [Glance, 1971 Reprinted by permission of the Bell Telephone Laboratories Inc. Copyright 1971. The American Telephone and Telegraph Company]

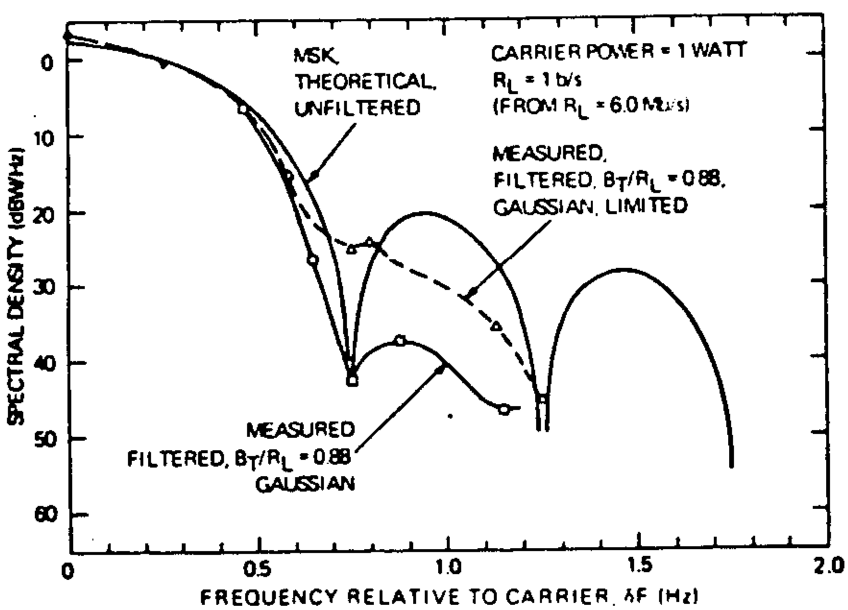

Figure 13. Spectrum of MSK

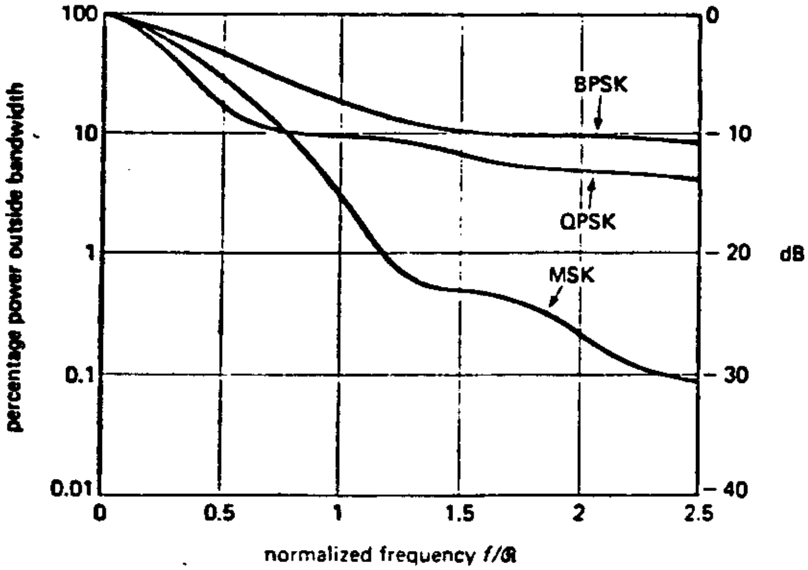

FIG 14. Integrated power spectra for PSK, QPSK and MSK; frequency is normalized to the bit rate ℛ.